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# FINITE ELEMENT RESULTS OF PRESSURIZED THICK TUBES BASED ON TWO ELASTIC-PLASTIC MATERIAL MODELS

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## TABLE OF CONTENTS

	<u>Page</u>
INTRODUCTION	1
ELASTIC-PLASTIC THEORY	2
FINITE ELEMENT FORMULATION	4
THICK TUBES	5
REFERENCES	9

## LIST OF ILLUSTRATIONS

1. Stress-Strain Curve for a High Strength Steel.	11
2. Boundary Displacements as Functions of Pressure History ( $b/a = 2$ ).	12
3. Hoop Stress Distribution at Different Stages of Loading ( $b/a = 2$ ).	13
4. Distribution of Radial and Axial Stresses and Equivalent Plastic Strain after Complete Unloading ( $b/a = 2$ ).	14
5. Boundary Displacements as Functions of Pressure History ( $b/a = 3$ ).	15
6. Stresses Near the Bore as Functions of Pressure History ( $b/a = 3$ ).	16
7. Boundary Displacements as Functions of Pressure History ( $b/a = 4.63$ ).	17
8. Distribution of Hoop Stress in the Inner Half of Different Stages of Loading ( $b/a = 4.63$ ).	18
9. Distribution of Residual Radial and Axial Stresses in the Inner Half ( $b/a = 4.63$ ).	19

## INTRODUCTION

The problem of pressurized thick-walled tubes is of practical importance to pressure vessels and the autofrettage process of gun barrels. Many solutions for this problem have been reported over the last three decades (refs 1-7). This is a result of different mathematical methods, end conditions, and material models. Different assumptions for the material properties such as compressibility, yield criterion, flow rule, hardening rule, etc. can lead to many material models. A common feature in all earlier investigations is to introduce certain restrictive assumptions so as to simplify the mathematical analysis (refs 1-4). The recent development in numerical methods makes it possible to use a more realistic material model and to consider more general problems. Both the finite element method (ref 5) and the finite difference method (refs 6,7) have been used to solve the elastoplastic problems with different end conditions and more general loading conditions. The material model was based on the von Mises yield criterion, the Prandtl-Reuss flow theory, and the isotropic hardening rule.

The finite element method is more powerful and can be used to solve more general nonlinear problems (refs 8,9). Many finite element codes have been developed as seen in a recent survey paper (ref 10). The ADINA code, developed by K. J. Bathe, is a general purpose finite element program for Automatic Dynamic Incremental Nonlinear Analysis (ref 11). The standard version models the elastic-plastic behavior of metals by the use of the Mises yield criterion, the associated flow theory, and two strain-hardening rules -

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References are listed at the end of this report.

isotropic and kinematic. Both hardening models were limited to linear hardening in our first version acquired in 1981. The multi-linear option was allowed in our second version one year later. This report shows an application of the ADINA code to our pressurized thick tube problems. A multi-linear stress-strain curve is used in both material models and thick tubes of different wall ratios are considered. The numerical results together with a brief summary of the elastic-plastic theory, finite element formulation are presented below with emphasis on the basic assumptions used. More detailed theoretical information can be found in a forthcoming report (ref 12).

#### ELASTIC-PLASTIC THEORY

In elastic-plastic analysis the material behavior is described using three properties in addition to the elastic stress-strain relations, namely a yield criterion, a flow rule, and a hardening rule.

The initial and subsequent yield condition for isothermal kinematic or isotropic hardening can be written as

$$f(\sigma_{ij} - \alpha_{ij}) - \sigma(\int d\epsilon^p) = 0 \quad (1)$$

where  $\sigma_{ij}$  is the stress tensor,  $\alpha_{ij}$  is a tensor denoting the translation of the yield surface,  $f$  is the yield function, and  $\sigma(\int d\epsilon^p)$  represents the dependence of the yield stress on the accumulated increments of effective plastic-strain. The von Mises yield function for kinematic hardening is

$$f = \left[ \frac{3}{2} (s_{ij} - \alpha_{ij})(s_{ij} - \alpha_{ij}) \right]^{1/2} \quad (2)$$

where

$$s_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}$$

and

$$\alpha_{ij} = 0 \text{ for isotropic hardening} \quad (3)$$

Restricting the analysis to associated flow rules, the plastic strain increment  $d\epsilon_{ij}^p$  is derivable from the plastic potential function  $f$  by

$$d\epsilon_{ij}^p = q_{ij} d\lambda \text{ and } q_{ij} = \partial f / \partial \sigma_{ij} \quad (4)$$

where  $d\lambda$  is a scalar to be determined.

During active plastic deformation the yield function must be satisfied continuously, so that the consistency condition is

$$(d\sigma_{ij} - d\alpha_{ij}) \partial f / \partial \sigma_{ij} = 0 \quad (5)$$

The original kinematic hardening concept was Prager's rule (ref 13) that

$$d\alpha_{ij} = \left(\frac{2}{3} H'\right) d\epsilon_{ij}^p \text{ and } H' = d\sigma / d\epsilon^p \quad (6)$$

Prager's rule was used in the ADINA formulation although its modification by Ziegler (ref 14) is more popular. Equations (1) through (6) are the basic equations of the elastic-plastic theory. In addition, we need the elastic stress-strain relation

$$d\sigma_{ij} = E_{ijmn} (d\epsilon_{mn} - d\epsilon_{mn}^p) \quad (7a)$$

where  $E_{ijmn}$  is the elastic constitutive tensor. If the material is initially isotropic, then

$$E_{ijmn} = \frac{E}{1+\nu} [\delta_{im} \delta_{jn} + \frac{\nu}{1-2\nu} \delta_{ij} \delta_{mn}] \quad (7b)$$

where  $E$  and  $\nu$  are the Young's modulus and Poisson's ratio, respectively.



Using the basic equations (1) to (7), we can obtain the incremental stress-strain relation for elastic-plastic material models

$$d\sigma_{ij} = D_{ijmn} d\epsilon_{mn} \quad (8a)$$

where

$$D_{ijmn} = E_{ijmn} - \frac{E_{ijtu} q_{tu} q_{vw} E_{vwmn}}{H' + q_{kl} E_{klrs} q_{rs}} \quad (8b)$$

This constitutive relation holds for the combined isotropic-kinematic hardening model. For the special cases using Eqs. (2) to (4), we have

$$\text{isotropic hardening: } q_{ij} = 3s_{ij}/(2\sigma) \quad (9a)$$

$$\text{kinematic hardening: } q_{ij} = 3(s_{ij} - \alpha_{ij})/(2\sigma) \quad (9b)$$

#### FINITE ELEMENT FORMULATION

The finite element formulation used in the ADINA code is very general and large strain dynamic analysis has been considered (refs 11,12). Since the present problem requires only a small strain static analysis, a very brief summary of the special formulation is presented here. The geometry of the body is discretized by two-dimensional 8-nodes isoparametric elements. The coordinates and displacements are interpolated by the same shape functions  $N_i$ ,  $i = 1$  to 8, i.e.,

$$x = N_i \bar{x}_i, \quad u = N_i \bar{u}_i, \quad \text{etc.} \quad (10)$$

where  $\bar{x}_i, \bar{y}_i, \bar{u}_i, \bar{v}_i$  are the coordinates and displacements at the nodal points. The strain increments in elements can be obtained by differentiation and in matrix notation we have

$$\{\Delta\epsilon\} = [B]\{\Delta U\} \quad \text{and} \quad [B] = [L][N] \quad (11)$$

where  $[L]$  is a linear differential operator and  $\{\Delta U\}$  is a vector of all nodal displacement increments in an element.

Once we know  $[D]$  and  $[B]$ , we can compute the element stiffness matrix by

$$[K] = \int_v [B]^T [D] [B] d(\text{vol}) \quad (12)$$

To carry out numerical integration, we express all matrices and volume element in terms of local coordinates and evaluate them at integration stations with the aid of Gauss quadrature formulae. For double summation we use either (2x2) or (3x3) points in a rectangle. This finite element formulation is based on displacements so the kinematic equations and constitutive equations are satisfied locally. The principle of virtual displacements is used to express the equilibrium of the body in the current configuration. Since the principle is in integral form, we can sum all element contribution to the system.

#### THICK TUBES

Consider a long thick tube, internal radius  $a$ , and external radius  $b$ , which is subjected to internal pressure  $p$ . Thick tubes of different wall ratios are considered. The geometry of the tube is discretized by two-dimensional axisymmetric 8-nodes isoparametric elements along the radial direction. We use 10 elements for smaller wall ratios ( $b/a = 1.5$  and  $2.0$ ) and 20 elements for larger wall ratios ( $b/a = 3.0$  and  $4.63$ ). All elements are of equal size and 3x3 points are used in carrying out the numerical integration. The displacements at the nodal points and the stresses at the integration points are obtained as functions of loading history. At each stage of loading, we have  $N+1$  results for the displacements and  $3N$  results for the stresses where  $N$  is the number of elements used.

The common input data for both material models are  $E = 2.583 \times 10^7$  psi,  $\nu = 0.3$ , and 6 points on the uniaxial stress-strain curve, i.e.,  $(\sigma \text{ in Ksi}, \epsilon \text{ in } \%) = (155, 0.6), (167, 0.85), (172, 1.25), (177, 3.0), (181, 5), (181, 15)$ . These six points are chosen to give a piecewise linear representation to the actual stress-strain curve for a high strength steel as shown in Figure 1. The ADINA code allows a maximum of 7 points to represent two multi-linear hardening models (model number 8 and 9 for isotropic and kinematic hardening). These two hardening models are widely used because of their simplicity. Isotropic hardening is generally considered to be a suitable model for large plastic flows. Kinematic hardening is the simplest theory that can model the Bauschinger effect. If unloading does not occur, there is no difference between these two models. For unloading with reverse yielding, the finite element results based on these two models will be different.

The loading and unloading problems in thick tubes of different wall ratios have been analyzed using the ADINA code and two hardening models. The tubes of wall ratios 1.5 and 2.0 have been loaded to reach fully plastic state and then unloaded completely. No reverse yielding occurs during unloading for tubes with both wall ratios and the usual assumption of elastic unloading is justified on the basis of these two material models. The numerical results for the tube with  $b/a = 2$  are shown in Figures 2 through 4. Figure 2 shows the boundary displacements ( $U_a$  and  $U_b$ ) as functions of pressure history. We use 11 steps during loading and 2 steps during unloading. Figure 3 shows the hoop stress distribution at different  $t$  steps ( $t = 1, 6, 11, 13$ ) where  $t$  is a time-like parameter for the purpose of bookkeeping. Figure 4 shows the distributions of residual radial and axial stresses and equivalent plastic

strain. The residual stresses are considered to be elastic according to these two models. The unloading process may not be purely elastic if other models (ref 4) are used. Future work should search for a more realistic model including the Bauschinger effect in a high strength steel (ref 15). Experimental measurements, if available, should be used for comparison with numerical predictions.

The tube of wall ratio 3( $a = 1"$ ,  $b = 3"$ ) has also been loaded to reach fully plastic state and then unloaded completely. We use 11 steps during loading and 4 steps during unloading. Figure 5 shows the boundary displacements ( $U_a$  and  $U_b$ ) as functions of pressure history. The numerical results for the displacements during unloading are very close between the two models. However, there are noticeable differences in the size of reverse yielding and the stresses within a small zone near the bore. There are 60 stations along the radial direction at which the stresses are calculated. At the end of complete unloading, reverse yielding occurs at 3 or 7 stations near the bore according to isotropic or kinematic models, respectively. Figure 6 shows the stresses at a point near the bore as functions of pressure history. The differences between the two models for the hoop and axial stresses during unloading are not small as can be seen in the figure.

Finally, the autofrettage solution for a closed volume chemical "bomb", is obtained for a tube with  $a = 0.865"$  and  $b = 4.005"$ . The tube is loaded to  $p = 250$  Ksi in 10 steps and then unloaded completely in 5 steps. At maximum pressure, 26 of its 60 stations have become plastic. At the end of complete unloading, reverse yielding occurs at 2 or 5 stations near the bore according to isotropic or kinematic models, respectively. Figure 7 shows the boundary

displacements ( $U_a$  and  $U_b$ ) as functions of pressure history. There are small differences for the displacements during unloading based on two models. The results for the stresses within the inner half of the tube are presented in Figures 8 and 9. Figure 8 shows the hoop stress at different stages of loading and unloading. Three stages ( $t = 1, 10, 15$ ) represent the stage corresponding to initial yielding, maximum loading, and complete unloading, respectively. The differences for the hoop stresses during unloading based on two hardening models, are not small as can be seen in this figure. Figure 9 shows the differences for the axial and radial stresses within the inner half of the tube after complete unloading.

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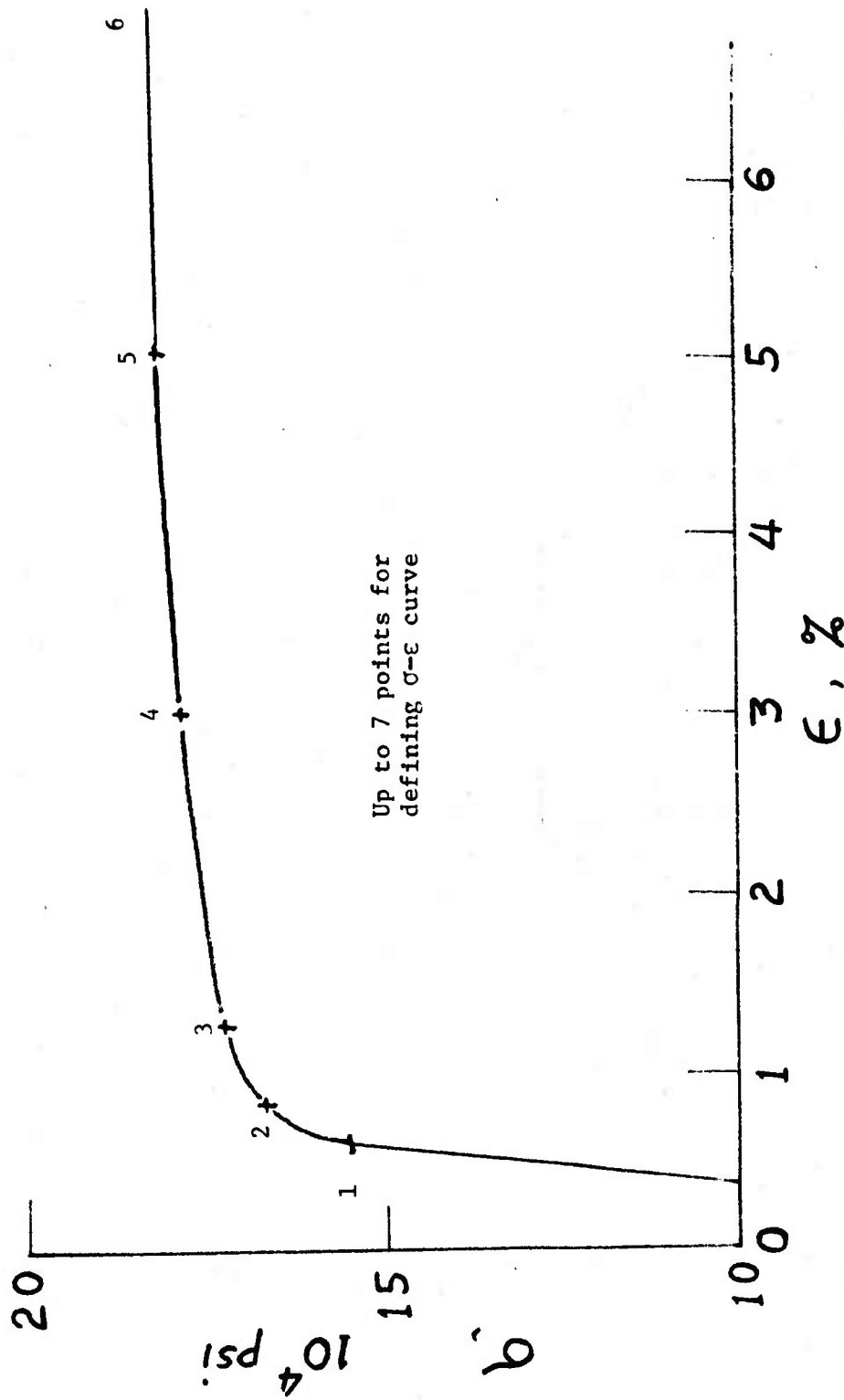


Figure 1. Stress-Strain Curve for a High Strength Steel.



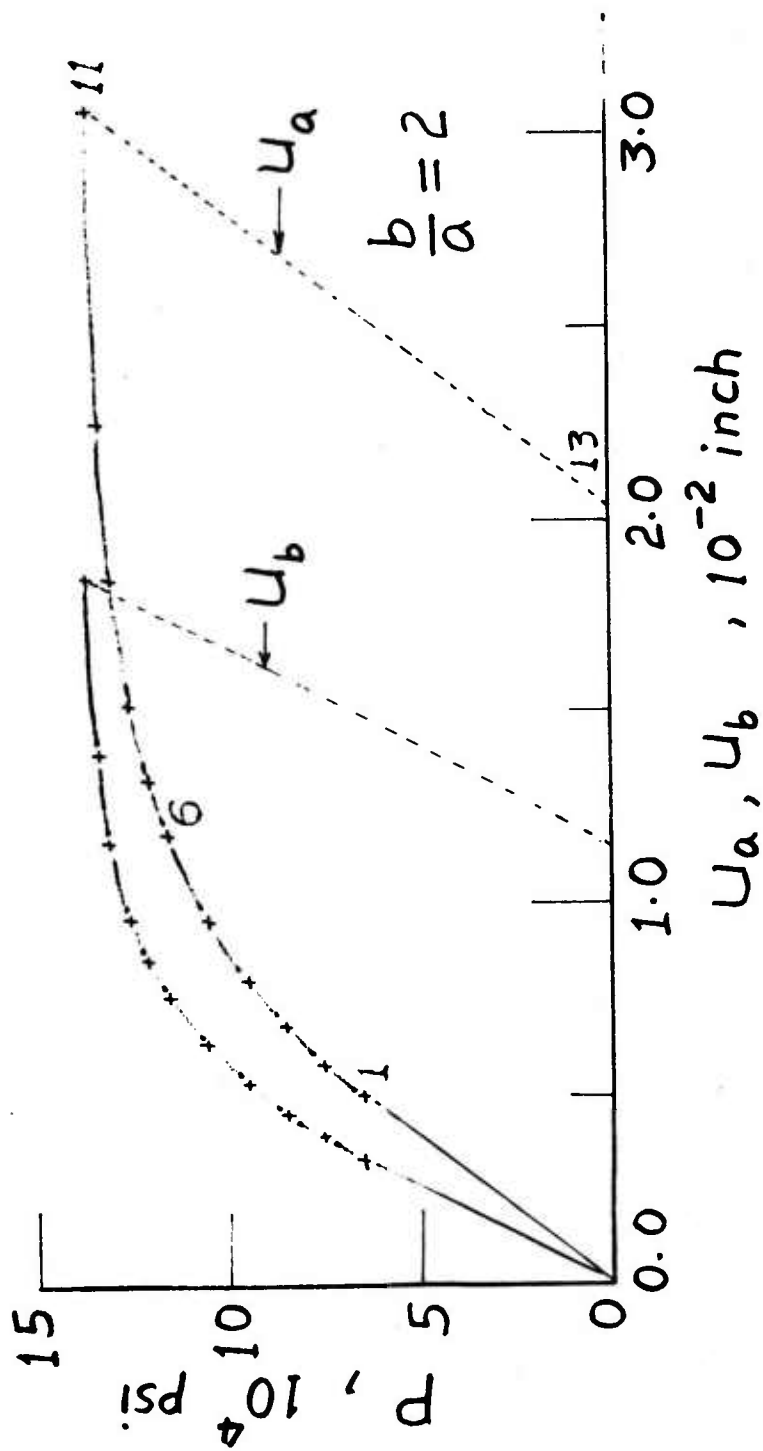


Figure 2. Boundary Displacements as Functions of Pressure History.

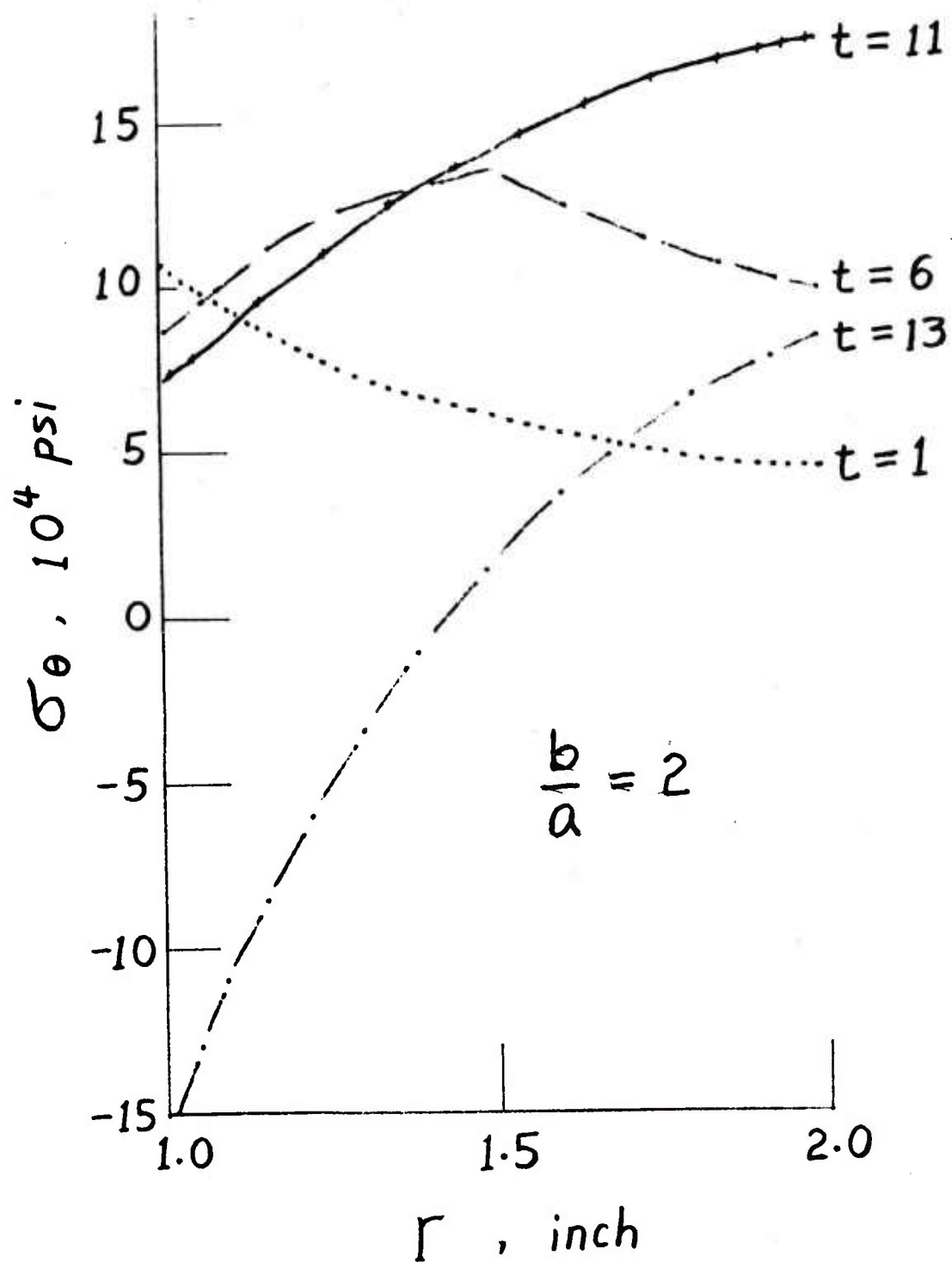


Figure 3. Hoop Stress Distribution at Different Stages of Loading.

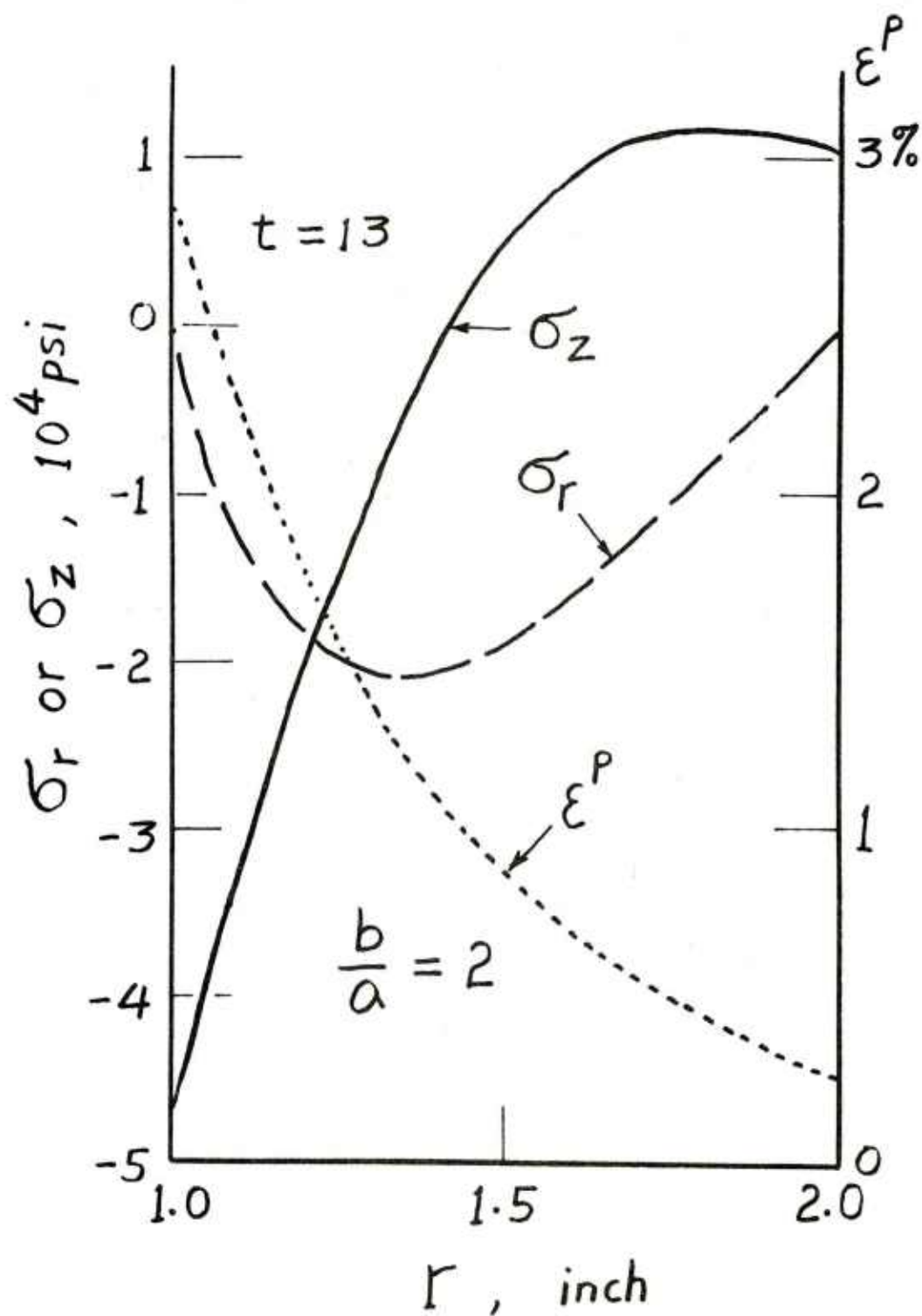


Figure 4. Distribution of Radial and Axial Stresses and Equivalent Plastic Strain After Complete Unloading.

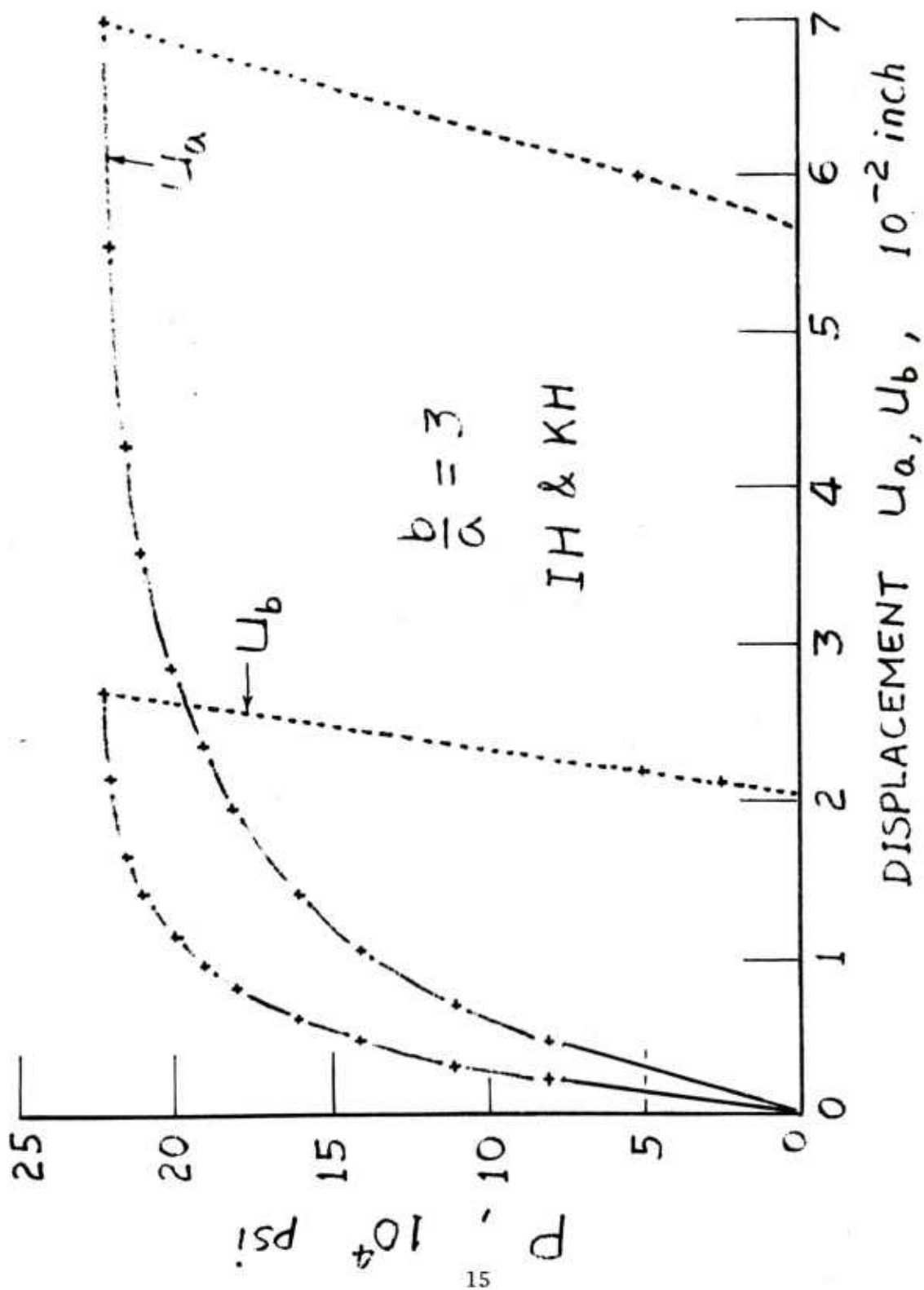


Figure 5. Boundary Displacements as Functions of Pressure History.

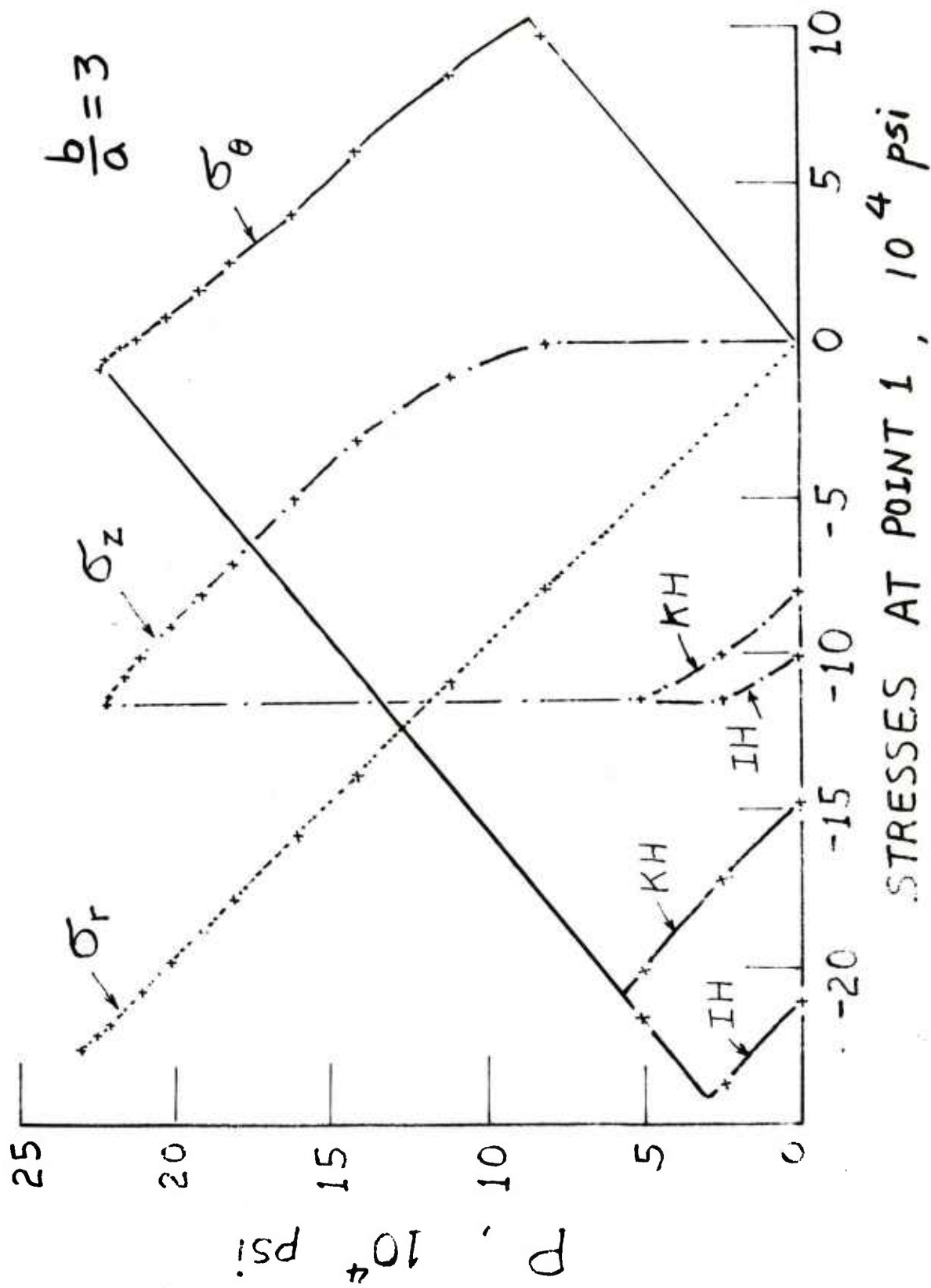


Figure 6. Stresses Near the Bore as Functions of Pressure History.

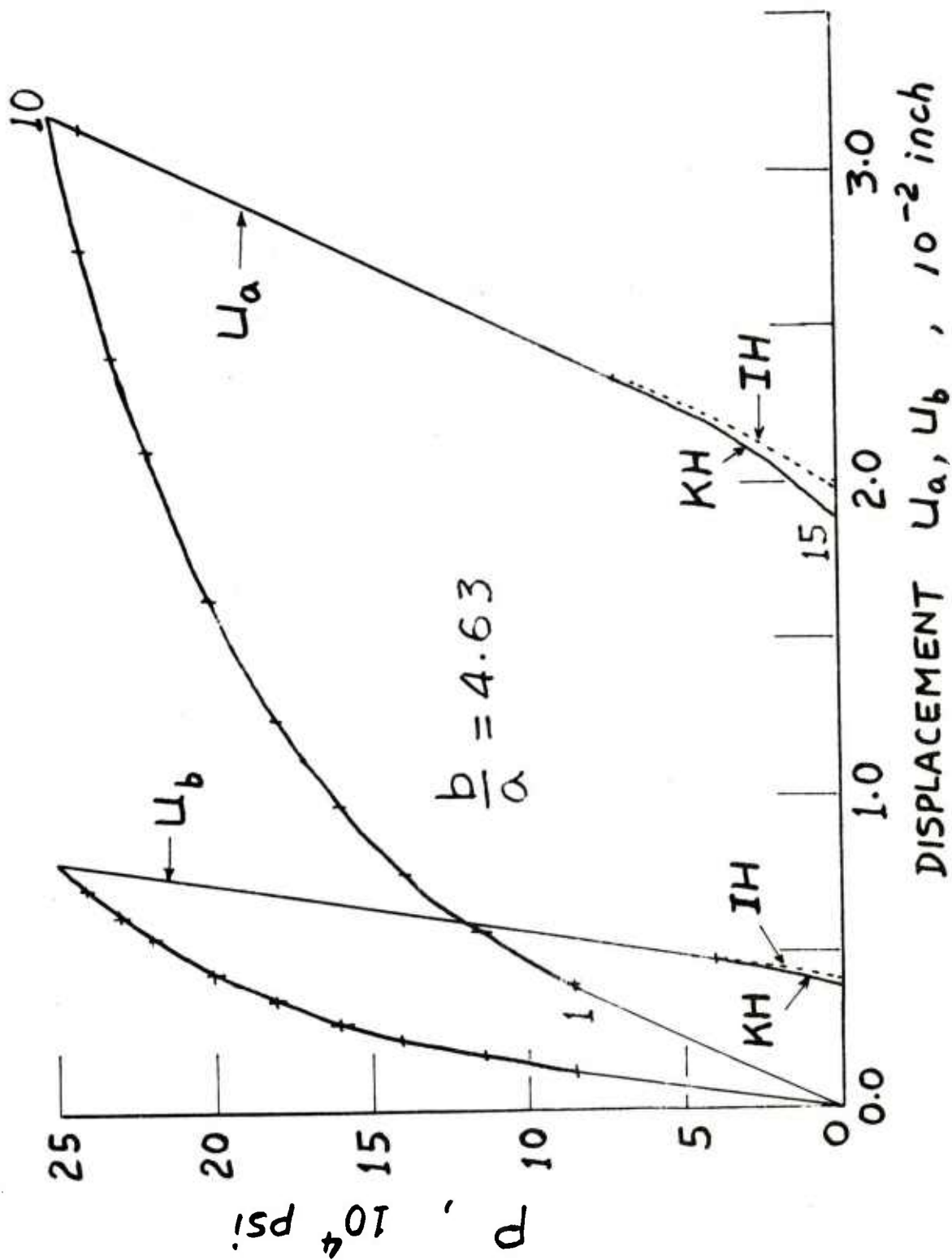


Figure 7. Boundary Displacements as Functions of Pressure History.

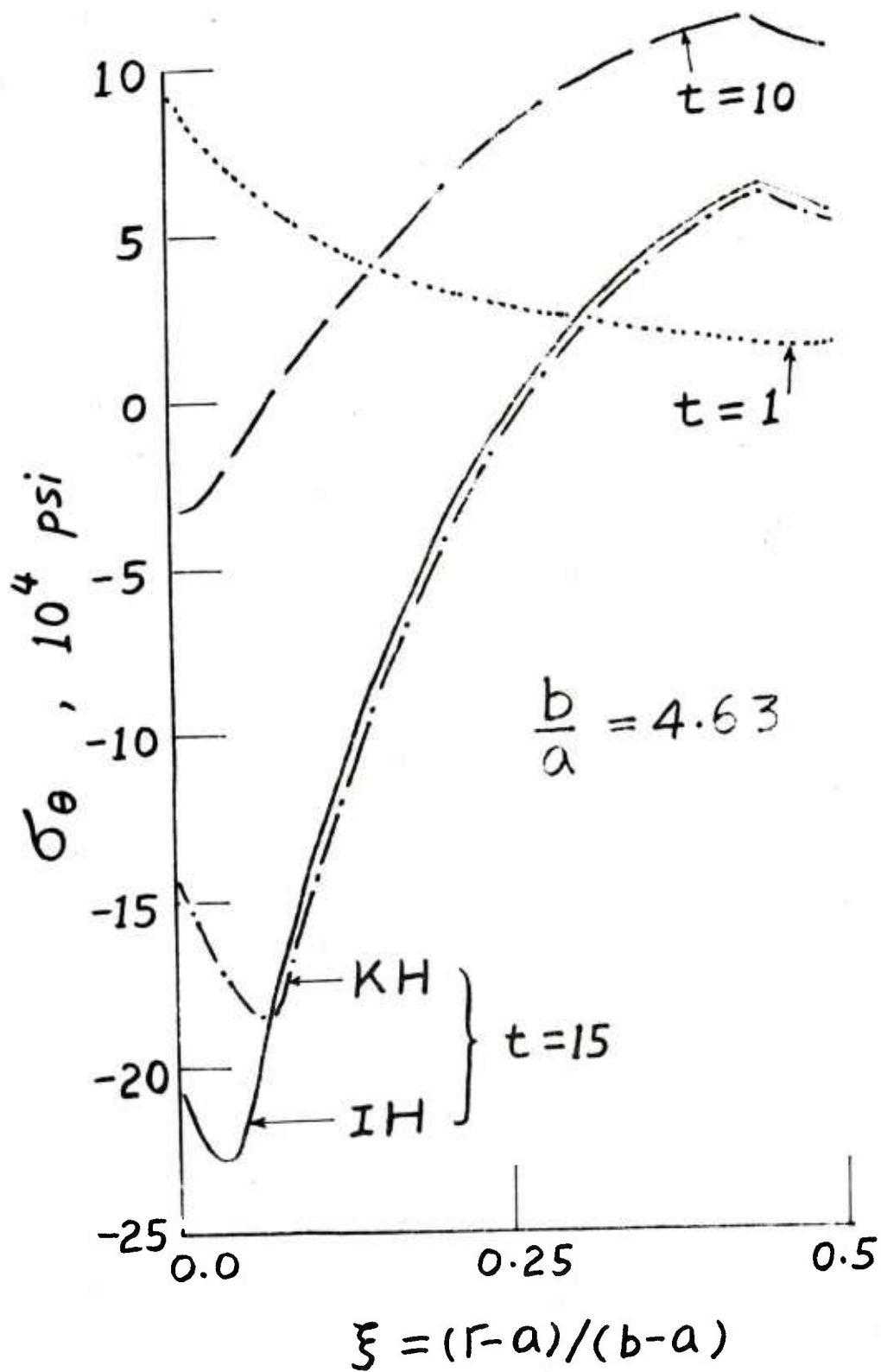


Figure 8. Distribution of Hoop Stress in the Inner Half at Different Stages of Loading.

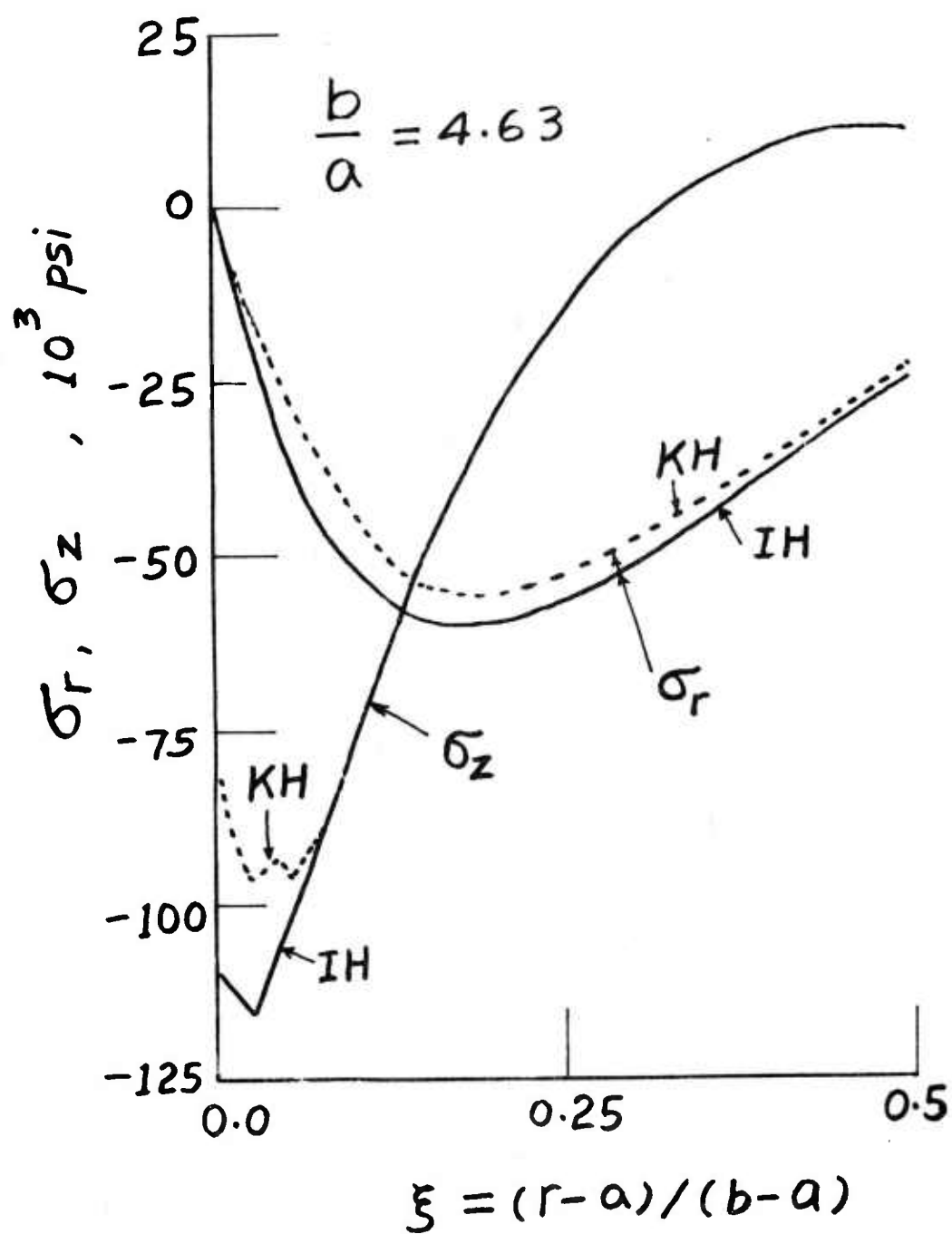


Figure 9. Distributions of Residual Radial and Axial Stresses in the Inner Half.



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